Erratum: Relativistic Brownian motion: From a microscopic binary collision model to the Langevin equation [Phys. Rev. E 74, 051106 (2006)]

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In Sec. II of this paper we have discussed how one can obtain a nonrelativistic Langevin equation (NRLE) from a simple microscopic collision model by means of several approximation steps. In the paragraph below Eqs. (15) it was stated that, given the fluctuation-dissipation theorem (15d), only the post-point discretization rule [1–4] yields a Maxwellian as the stationary distribution of the Brownian particle. This is correct, but for the post-point discretization rule the mean value of the fluctuating force $\xi(P,t)$ will be nonzero in general, i.e., $\langle \xi(P,t)\rangle \neq 0$. A vanishing mean value $\langle \xi(P,t)\rangle = 0$, as indicated in Eq. (15b), is obtained only if one adopts the Ito pre-point discretization rule [5–7]. Therefore, in order to make the dependence on the discretization rule more explicit, one should rewrite the NRLE (15a) in terms of an explicit multiplicative coupling (with post-point discretization), i.e.,

$$\dot{P} = -\nu_0(P)P + \sqrt{2D_0(P)}\,\bar{\xi}(t), \tag{15a}$$

where now $\bar{\xi}(t)$ is a normalized, momentum-*independent* Gaussian white noise, characterized by

$$\langle \bar{\xi}(t)\rangle = 0, \quad \langle \bar{\xi}(t)\bar{\xi}(s)\rangle = \delta(t-s).$$

In the limit where the Brownian particle is much heavier than the heat bath particles, the momentum-dependent noise amplitude $D_0(P)$ is determined by the fluctuation-dissipation theorem $D_0(P) = M\nu_0(P)kT$ with friction coefficient $\nu_0(P)$ given by Eq. (11). Then, in accordance with the above remarks, one finds $\langle \sqrt{2D_0(P)}\,\bar{\xi}(t)\rangle = 0$ only for the Ito stochastic integral interpretation [5–7], but $\langle \sqrt{2D_0(P)}\,\bar{\xi}(t)\rangle \neq 0$ for any other discretization rule [1–4].

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